Multi-dimensional Support Vector Regression in Electrical Capacitance Tomography

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Abstract — Electrical capacitance tomography (ECT) image reconstruction is a typical ill-posed problem. In this paper, an image reconstruction method based on multi-dimensional support vector regression (MSVR) with hyper-spherical insensitive zone is presented. Similar iterative re-weight least square (IRWLS) algorithm was used to simply the realization of MSVR to quickly build a nonlinear map between capacitance measurements and the permittivity distribution in image region. The proposed MSVR method was verified through typical flow patters image reconstruction. The results show that this method is an effective approach to solve image reconstruction for ECT, which is faster compared with the iterative methods and more accurate compared with the neural networks.

I. INTRODUCTION

Electrical capacitance tomography (ECT) works by placing electrodes around a domain of interest, measuring the change of the coupling capacitance between each pair, and reconstructing a tomography image from the measurements using image reconstruction algorithms. It provides a means of visualizing electrical permittivity distribution in a cross section. The advantages of ECT are that it is low cost, high speed, robust, non-intrusive, and non-invasive. The information provided by ECT enables improved monitoring and control of industrial processes [1].

Capacitance sensor is a soft field sensor. In addition, the number of possible independent measurements is very small compared with the number of pixels required for an acceptable image. That leads to the inverse problem is ill-posed [2]. Successful applications of ECT depend greatly on the precision and speed of the image reconstruction algorithm.

II. ECT IMAGE RECONSTRUCTION

There are two major computational aspects in the image reconstruction for ECT: the forward problem and the inverse problem. The forward problem in ECT determines the inter-electrode capacitances from the permittivity distribution. The inverse problem aims to determine the permittivity distribution from the measured capacitance data. The result is usually presented as a visual image, and hence this process is called image reconstruction. The electrical field inside an ECT sensor can be calculated using the Poisson equation by finite element method (FEM), which is given by

$$\varepsilon(x,y)\nabla^2\phi(x,y)+\nabla\varepsilon(x,y)\nabla\phi(x,y)=0 \tag{1}$$

where $\varepsilon(x,y)$ and $\phi(x,y)$ are the dielectric constant and the electrical potential distributions, respectively. The relationship between the capacitance and the permittivity distribution is governed by

$$c=\frac{Q}{V}=-\frac{1}{V}\iint_{\Gamma}\varepsilon(x,y)\nabla\phi(x,y)\mathrm{d}\Gamma \tag{2}$$

where Q is the charge, V the potential difference between two electrodes forming the capacitance and Γ the electrode surface. In ECT applications, the image reconstruction model can be simplified as:

$$C=SG \tag{3}$$

where C is the normalized capacitance vector, G is the normalize permittivity distribution, namely the picture pixel's gray-level vector, and S is the normalized sensitivity matrix.

Compared with neural networks, support vector machine (SVM) is a novel and powerful machine learning approach. It holds the advantages of good generalization, being insensitive to high dimension data and convergence to global optimum, so it can solve the intractable problems that neural networks meet, including the over-fitting, local minima, dimension curse etc. Fernando Pérez-Cruz firstly presented the theory of multidimensional support vector regression (MSVR) in 2002[3], which has a less complex structure and faster training speed with less samples than a single output SVR model. We have applied it to EEG source localization successfully in [4-5].

In this paper, we proposed a method based on MSVR with hyper-spherical insensitive zone to solve the image reconstruction problem in ECT.

III. IMAGE RECONSTRUCTION BASED ON MULTIDIMENSIONAL SUPPORT VECTOR REGRESSION

A MSVR has been recently proposed: given a labeled data set $\{(x_i, y_i)\}, i=1,\cdots,n$, with $x\in R^d$, $y\in R^k$. MSVR is defined as the mathematical estimation of y given x, which can be obtained by finding the regressor W and b than minimizes

$$L_p(W,b)=\frac{1}{2}\sum_{j=1}^{k}\left\|w^j\right\|^2+C\sum_{i=1}^{n}L(u_i) \tag{4}$$

where $u_i=\|e_i\|=\sqrt{(e_i^T e_i)}$, $e_i^T=y_i^T-\phi(x_i)W-b^T$

$$L(u)=\begin{cases}0 & u<\varepsilon\\ u^2-2u\varepsilon+\varepsilon^2 & u\geq\varepsilon\end{cases} \tag{5}$$

$L(u)$ is a quadratic epsilon-insensitive cost function, and $\phi(\cdot)$ is an nonlinear transformation to the feature space.

In MSVR, using the inner products between $\phi(\cdot)$, we only need to define a kernel function:

$K(x_i, x_j) = \phi^T(x_i)\phi(x_j)$ that has to fulfill Mercer Theorem, in our studies, we selected Gaussian function $K(x_i, x_j) = \exp(-\|x_i - x_j\|^2 / (2*\sigma 2))$ as the kernel function.

An iterative reweighted least square (IRWLS) procedure is presented. To construct an IRWLS procedure, we modify (4) using a first order Taylor expansion of L(u) at u_i^m

$$L_p'(W,b) = \frac{1}{2}\|w^j\|^2 + C\left(\sum_{i=1}^{n} L(u_i^m) + \frac{dL(u)}{du}\bigg|_{u_i^m} \frac{(e_i^m)^T}{u_i^m}[e_i - e_i^m]\right) \quad (6)$$

In order to obtain w^s and b^s , which is the solution to (6), equating its gradient to zero, which can be expressed as a linear system of equations

$$\begin{bmatrix} \Phi^T D_a \Phi + I & \Phi^T a \\ a^T \Phi & 1^T a \end{bmatrix} \begin{bmatrix} w^j \\ b^j \end{bmatrix} = \begin{bmatrix} \Phi^T D_a y^j \\ a^T y^j \end{bmatrix} \quad (7)$$

where

$\Phi = [\phi(x_1), \cdots, \phi(x_n)]^T$, $a = [a_1, \cdots, a_n]^T$, $(D_a)_{ij} = a_i \delta(i-j)$, $j = 1, \cdots, k$ and $y^j = [y_{1j}, \cdots, y_{nj}]^T$.

IV. EXPERIMENTAL RESULTS

To verify the proposed image reconstruction algorithm, a model was constructed and the forward problem solution for ECT was obtained by FEM. The training sets were obtained by using finite element software developed by the authors. The image area is divided into 804 elements. 240 samples of eight flow regimes were obtained. Each kind of flow regimes is 30. We randomly chose 120 samples to constitute a training set, and the others as a test set.

Instead of the quadratic programming, IRWLS procedure is assessed for training the MSVR. The training parameter C=200, the RBF kernel function with $\sigma = 1.0$, and the value $\varepsilon = 0.001$ is given for the same error range in the hyper spherical insensitive zone. The results show that the algorithm is excellent and able to distinguish all kind flow regimes correctly.

In all test examples, we randomly chose one sample of each kind. Identification results of reconstructed images are shown in Table I. In order to evaluate the correctness of reconstructed images, we define the accuracy rate, which is defined by $C = N_{correct} / N_{all}$, where $N_{correct}$ is the number of correct pixels in reconstructed image. N_{all} is the number of all pixels, which is 804. The accuracy rate of six flow regimes is shown in Table II.

TABLE I
SIMUALITON RESULTS FOR TYPICAL MEDIAM DISTRIBUTION

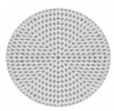
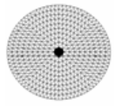
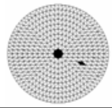
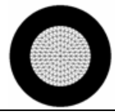
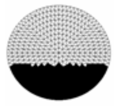
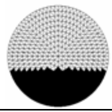
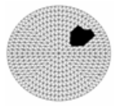
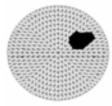
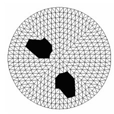
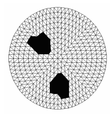
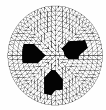
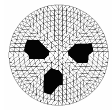
Flow patters	Permittivity distribution	Reconstructed Image
empty-pipe flow		
full-pipe flow		
core flow		
annular flow		
stratified flow		
single bubbly flow		
two bubblies flow		
three bubblies flow		

TABLE II
ACCURACY RATE OF EIGHT FLOW REGIMES

kind of examples	Accuracy rate
empty-pipe flow	96.6%
full-pipe flow	97.4%
core flow	98.2%
annular flow	95.3%
stratified flow	96.5%
single bubbly flow	97.1%
two bubblies low	94.4%
three bubblies flow	95.7%

V. REFERENCES

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